The wonderful world of chordal graphs

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Abstract

Chordal graphs are perhaps the second most interesting and important family of graphs – after trees and before planar graphs. Their fame is due to their beautiful and classical characterizations, their diverse mathematical properties, and their numerous applications in combinatorial optimization, linear algebra, statistics, constraint programming, relational databases, signal processing, machine learning, and techniques for exploiting sparsity in large positive semidefinite matrices. Originally known as rigid circuit graphs and later triangulated graphs, they are defined by the property that for every cycle of length greater than or equal to four there is an edge (called a chord) connecting two vertices that are not consecutive on the cycle.

In algorithmic graph theory, chordal graphs were one of the earliest families for which structural properties fundamentally help in solving hard problems efficiently, including the coloring, clique, independent set, and clique cover problems. They lead to researchers looking carefully at the tree structure of graphs and hypergraphs, and developing the notion of treewidth and partial k-trees, which have many algorithmic consequences. Lexicographic breadth first search (LexBFS) and maximum cardinality search (MCS) have their origins in recognizing chordal graphs. A large hierarchy of graph classes has been built around chordal graphs, each with its own characterizing properties and applications. In this lecture, we will present some of the significant developments involving chordal graphs and related families of graphs.